How do Parameters Affect the Dynamic Stability of a Mathematical Population Model?

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Abstract

This project's idea was to test whether the population dynamics depend on three model parameters: α, β, δ. I designed codes using MAPLE software to create a bifurcation diagram for this model to test whether the system becomes chaotic as we change only one parameter at a time. I kept α and δ constant, designed MAPLE codes and plotted the bifurcation diagram to test how β affects the population dynamics. Then, I chose one δ and three β constants (equilibrium, period 2 and chaos ranges) to test how α affects the population dynamics. Lastly, I chose one α and three β constants to test how δ affects the population dynamics.

When β increases, the model demonstrates bifurcation, periods, chaos and periodic windows. When β is in the equilibrium range and α increases, the model approaches an equilibrium. When β is in period 2 and the chaotic range with α increasing, the model demonstrates more complex dynamics with earlier and shorter bifurcation, periods, chaos and periodic windows. When β is in the equilibrium range and δ increases, the model approaches an equilibrium. When β is in period 2 range and δ increases, the model demonstrates bifurcation and then equilibrium. When β is in the chaotic range and δ increases, the model demonstrates chaos, period 4, period 2 and then equilibrium.

In conclusion, increased rate of aggression between adults and juveniles or increased birth rate destabilizes the population dynamics. Increased rate of aggression between juveniles and juveniles stabilizes the population dynamics.

Rationale

Mathematical models can be used to predict the population of species considering both reproduction & aggression. In the real world these are extremely important, for example mathematical model could be used to predict the population of novel 2019 coronavirus. Different parameters of the model might affect the dynamic stability differently. The mathematical model I used for this project is:

$$x_{n+1} = \frac{\alpha\beta xn e^{-\beta xn}}{1 + \frac{\alpha\delta}{2\beta}(1 - e^{-\beta xn})}$$

x_{n+1}: The population in year n+1

x_n: The population in year n

β: The constant per unit rate adult species attack juveniles

δ: The constant per unit rate juvenile species attack juveniles

α: Birth rate of the population

Hypothesis

In a population mathematical model consists of adults and juveniles, and adults and juveniles attack juveniles, juveniles produce and adults die after 1 year, then the population dynamics depend on differently model parameters significantly.

Methodology

I designed codes using MAPLE software to create a bifurcation diagram for this model to test whether the system becomes chaotic as we change only one parameter at a time. I kept α and δ constant, designed MAPLE codes and plotted the bifurcation diagram to test how β affects the population dynamics. Then, I chose one δ and three β constants (equilibrium [β=1], period 2 [β=4.5] and chaos [β=9]) to test how α affects the population dynamics. Lastly, I chose one α and three β constants (equilibrium [β=1], period 2 [β=4.5] and chaos [β=9]) to test how δ affects the population dynamics.

Example MAPLE codes with β as variable.

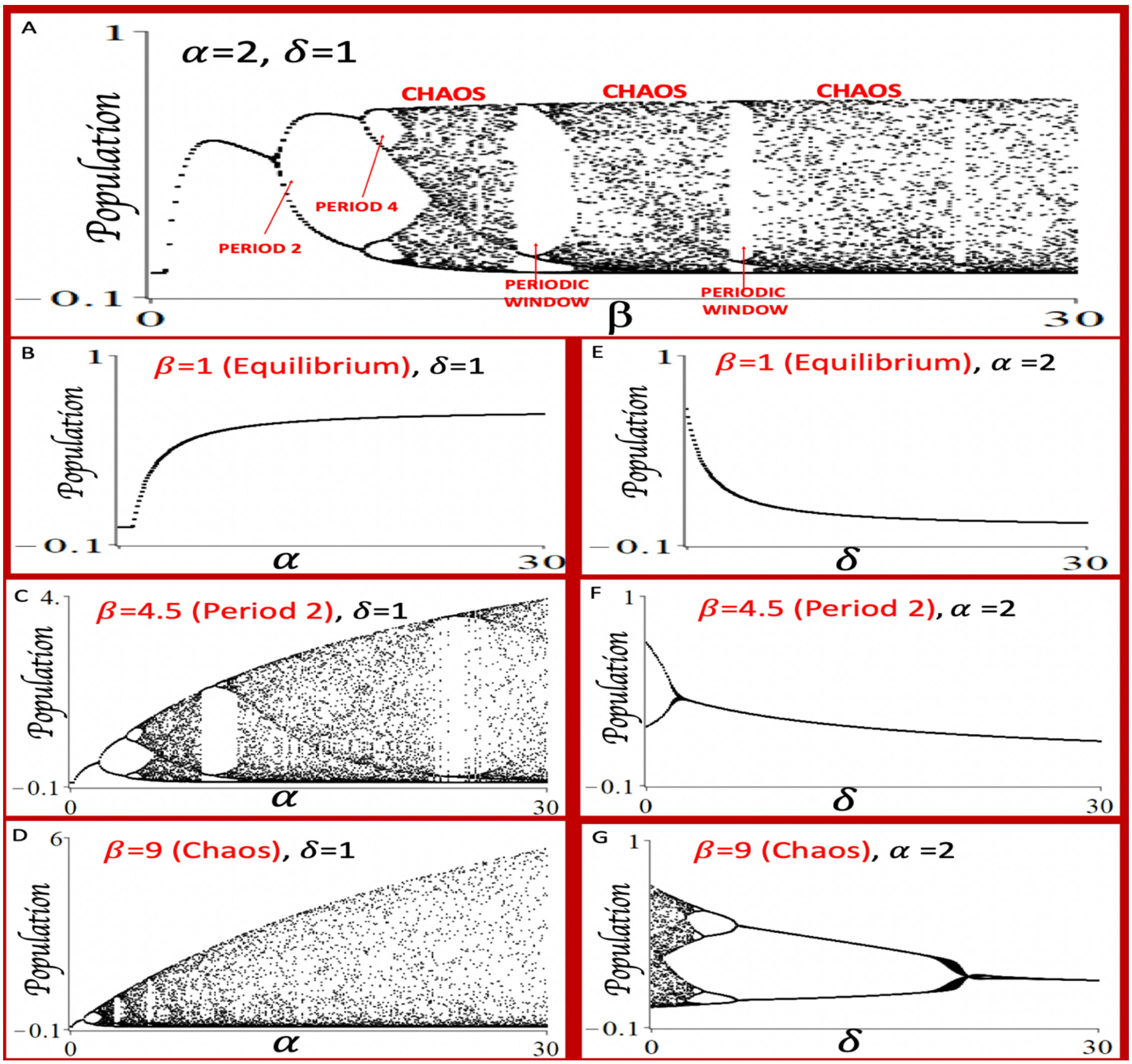
```
> with(plots) :
 imax := 80 : jmax := 300 : step := 0.1 :
ll := array(0 ..10000) : pp := array(0 ..10000) : xx := array(0 ..10000, 0 ..10000) :

for j from 0 to jmax do
xx[j, 0] := 0.2 :
 for i from 0 to imax do
xx[j, i + 1] := (2·xx[j, i]·(step·j)·(step·j)·exp(-(step·j)·xx[j, i])) / (step·j + 1 - exp(-(step·j)·xx[j, i])) :od:
ll[j] := [[(step·j), xx[j, n]]$n = 40 ..imax] :od:
LL := [seq(ll[j], j = 0 ..jmax)] :
with(plots) :
P1 := plot(LL, x = 0 ..30, y = -0.1 ..1, style = point, symbol = point, color = black, tickmarks = [2, 2]) :
# Plot the bifurcation diagram #
display({P1}, labels = [``, `population`], color = black, tickmarks = [2, 2], font = [TIMES, ROMAN, 25], axes = FRAMED);
```

Results and Analysis

- When β increases beyond the first bifurcation point, the period is doubled for bifurcation. The sequence of doubling ends when β reaches a certain value, where the system becomes chaotic. After the system becomes chaotic, there are regions where the system returns to periodic behavior −Periodic Windows (A).

- When β is in the equilibrium range and α increases, the model finally approaches an equilibrium (constant solution) (B).
- When β is in the period 2 range and α increases, the model demonstrates bifurcation, periods, chaos and periodic windows (C).
- When β is in the chaotic range and α increases, the model demonstrates more complex dynamics with earlier and shorter bifurcation, periods, chaos and periodic windows (D).

- When β is in the equilibrium range and δ increases, the model approaches an equilibrium (E).
- When β is in the period 2 range and δ increases, the model demonstrates bifurcation and then equilibrium (F).
- When β is in the chaotic range and δ increases, the model demonstrates chaos, period 4, period 2 and then equilibrium (G).



Conclusions

Increased rate of aggression between adults and juveniles or increased birth rate destabilizes the population dynamics and demonstrates chaotic characteristics with periodic windows in between in this model. Increased rate of aggression between juveniles and juveniles stabilizes the population dynamics. Chaos can cause the population to run a higher risk of extinction and make the population become out of control due to unpredictability. We can introduce factors to control the chaos and bring equilibrium to the system or introduce factors to induce chaos to the system depending on whether we want the system to survive. Mathematical models can be used to predict dynamic behaviors of virus growth, so that we can make the system chaotic and eliminate the virus from human.

References

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2. Dynamical Systems with Applications using MAPLE by Stephen Lynch Birkhauser Boston 2001.